Designing Fuzzy Logic Control for the BLDC Motor Based on Airfoil Bearing

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Abstract

Due to removing the mechanical brushes in BLDC motors, BLDC motors are widely used in many industrial applications. Therefore, they are maintenance-free and their control is easy to design, especially for high-speed applications. For example, eco-friendly vacuum cleaners prevent atopic dermatitis, allergic rhinitis, and asthma. Other applications are Micro Gas turbines or compressors with small impellers due to miniaturization, BLDC motors must rotate at very high speeds to maintain the compressor's compression ratio. Generally, airfoil bearings should be used instead of ball bearings because of the friction in high-speed motors. Unfortunately, the characteristics of airfoil bearings depend on rotational speed. In this work, a BLDC motor with an airfoil bearing is controlled by a PID controller, this work analyzed the BLDC SYSTEM to determine the PID coefficient using the feedback method. The proposed controller Fuzzy Logic is used for adaptive control. In addition, the controller of BLDC motors combines auto-tuning and self-tuning technology. The results demonstrate that the proposed method gives efficient control by reducing the settling time and maximum peek overshot. The designed controller for the airfoil-bearing BLDC motor has a good performance.

Keywords: BLDC motor, PID controller, Fuzzy logic controllers, very high-speed Motors.

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1. Introduction

BLDC motors are used in many compressor industries such as industrial air compressors, gas turbines, large turbo chillers, and high-pressure blowers [1]. In recent years, the shipment volume of industrial compressors produced with domestic technology has increased [2]. Currently, most industrial compressors are rated at 100 horsepower or more. Compressors over 100 horsepower are expensive to install and take up a lot of space. Miniaturization and lightweight manufacturing while maintaining the compression ratio of the compressor is one way to increase competitiveness in the industrial compressor market. Compressor miniaturization and weight reduction mean that the size of the impeller is also reduced. Since the flow rate decreases as the impeller becomes smaller, the BLDC motor must rotate at a very high speed to maintain the existing compression ratio. One of the important things to operate a BLDC motor at high speed is the bearing system that supports the rotating shaft [3]. Bearings used in BLDC motors include ball bearings, magnetic bearings, and airfoil bearings. Ball bearings have poor durability due to friction

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with the rotating shaft, so there is a limit to the rotation of the motor at ultra-high speed. In addition, due to friction with the rotating shaft, an oil system is required and maintenance is expensive. The magnetic bearing has the disadvantage of an inherently unstable system that leads the controller to be a very complicated and high cost [4].

However, airfoil bearings can be rotated at ultra-high speeds and have high durability because air flows in between the bearing and the rotating shaft at a certain speed and can be protected from friction [5]. In addition, since the airfoil bearing and the rotating shaft do not come into contact, the cost of the oil system can be reduced and the system is simpler than the magnetic bearing. However, airfoil bearings have a disadvantage in that the durability of airfoil bearings deteriorates when initial driving is repeated because friction is high before an air layer is formed between the bearing and the rotating shaft at low speeds. In this paper, PID control, which is generally applied to motor control, was used to control a BLDC motor equipped with an airfoil bearing [6].

The Ziegler-Nichols' closed loop method, an auto-tuning method, was used to analyze the system characteristics and determine the PID coefficient values. Ziegler-Nichols' closed loop method is a method to determine the PID coefficient value by identifying the characteristics of the system from continuous output vibration [7]. In this paper, output vibration was generated by using a feedback method that can limit the plant output and thus ensure system stability. Since the airfoil bearing with the BLDC motor operates at a very high speed with an air layer formed between the bearing and the rotating shaft from a certain speed, the feedback reference point was set to a section above a certain speed. In addition, since the load torque and system characteristics of the motor vary rapidly according to the speed when operated at ultra-high speed, a fuzzy controller, one of the self-tuning methods, is combined to improve the system response and stable control for the changing load torque. The rule base of the fuzzy controller was set in consideration of the response characteristics of the system response to changes in the PID coefficient value for each control technique [8].

2. The ultra-high-speed brushless DC motor

The ultra-high-speed brushless DC motor consists of a motor part that generates rotational force, a shaft system (bearing) that transmits motor power, and a driver circuit for driving and control [9]. Recently, diseases such as atopic dermatitis, allergic rhinitis, and asthma are rapidly increasing [10]. This technology development is a high-speed motor and driver development technology, which is a key part used in eco-friendly vacuum cleaners to prevent so-called civilization diseases. Motors for vacuum cleaners can be largely divided into DC and AC modules according to the type of electrical input source of the motor, which is the source of generating rotational force [11]. and use, and production are rapidly growing. As for the clear way to solve this problem, it is considered that the best way so far is to eliminate the reality or the problem causing factor that modern medicine cannot suggest as shown in Fig. 1. In order to eliminate microorganisms such as house ticks, which are pointed out as the cause of these diseases, a vacuum cleaner with strong suction power is required, but for home use, it is limited in size, so a method of increasing the rotation speed is used. High-efficiency/high-speed single-phase/three-phase induction motors applied to existing household vacuum cleaners have already reached the limit, so it is urgent to develop high-speed, high-efficiency BLDC motors for energy saving [12].

High-speed, high-efficiency BLDC motor electromagnetic system designer/analyst is the driving principle of a BLDC motor. The brushless DC motor has a structure relative to that of a general permanent magnet DC motor. The permanent magnets in the DC motor are attached to the stator to provide the magnet field, and the armature winding is located on the rotor. In contrast, the brushless DC motor has a structure of a permanent magnet synchronous motor in which a permanent magnet is attached to a rotor and an armature winding is located on a stator [13]. In a general DC motor, terminals of the armature winding are connected to the commutator for continuous rotation of the rotor, and current flows through the armature winding through contact with a brush to which an external voltage is applied. On the other hand, in a brushless DC motor, an inverter is used instead of a mechanical brush and commutator to sequentially apply a square wave excitation current to the armature windings of the stator to generate rotational force.

Fig. 1 Composition of a DC system.

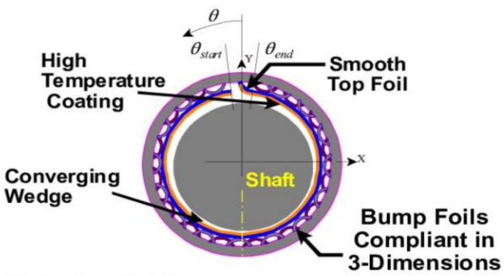

Fig. 2 Cross section of a DC motor with 2 brushes and 3 commutator pieces

3. The Technology of Air foil bearings

In this section, three aspects have been covered as the following:

3.1 Air Foil Bearings

An air foil bearing contains one or more foils between chambers and bearing housing and is a hydrodynamic bearing in which pressure is generated automatically (self-acting) in the gap of the bearing as the shaft rotates. It is a type of bearing. Fig. 2 shows the basic structure of an air foil bearing. Air foil bearings are largely composed of an elastic structure, a top foil, and an air oil film [14]. Looking at the driving process, since the top foil serves to form a lubrication gap between the shaft and the shaft, when dynamic air is formed in the lubrication gap as the shaft rotates, the shaft rises from the top foil and rotates using dynamic air as a lubricating medium. will do at this time, the top foil is supported by an elastic structure that is deformed in the presence of compliance. Since the elastic structure supports the rotating shaft by acting like a spring, the top foil surrounding the rotating shaft can maintain a minimal lubrication gap over a wider range than a general air bearing, and thus, stable rotation is possible. In this paper, a bump-shaped foil is used as an elastic structure. The material of the foil was copper coating was applied to the bump foil to increase the Coulomb damping. The upper surface of the top foil was coated with MoS2 to reduce the driving torque during initial operation [15].

3.2 Disadvantages of Air Foil Bearings

The damping of an air foil bearing is largely composed of damping in the air film, Coulomb damping due to friction between the top foil and bump foil, and Coulomb damping due to friction between the bump foil and bearing sleeve. As the stiffness increases, the damping decreases relatively rapidly, so that the damping of the entire bearing is dominated by the damping of the elastic structure. Therefore, when the damping of the elastic structure installed in the air foil bearing is small, the damping decreases as the eccentricity increases, unlike a general journal bearing. Considering that the working area of foil bearings usually performs well at large eccentricities, this is a disadvantage of foil bearings.

3.3 Viscoelastic air foil bearings

Viscoelastic air foil bearing is a foil bearing that improves the damping of the elastic structure compared to general foil bearings by additionally inserting a viscoelastic Z τ m and τ Lo are constant torque is supplied to the motor, J is the moment of inertia. The speed increases steadily, and then it operates in a steady state as an air layer is formed between the rotating shaft and the airfoil bearing from the point of a specific speed ωfo. When operating in a steady state, the load torque rapidly decreases to τ and the torque generated in the motor becomes Equation 1 From point ωf, the velocity increases rapidly as displayed in Fig. 3.

$$\tau_m - \tau_{Lf} = J d\omega_f / dt \quad (1)$$

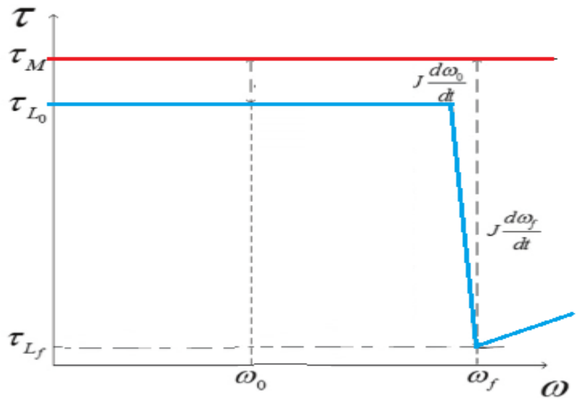

Fig. 3 The speed/torque characteristics of BLDC motor

5. Airfoil Bearing BLDC Motors Control

In this paper, the speed of the BLDC motor with airfoil bearing is controlled using the PID controller. When the target speed is determined, the target current is determined by the speed PID controller, and the torque of the motor is controlled by the current PID controller. The proposed design of the robust control using FLC to tune the parameters of PID automatically throws changing of speed and torque. Fig. 4 shows the block diagram of the system composed of the speed PID controller and the current PID controller.

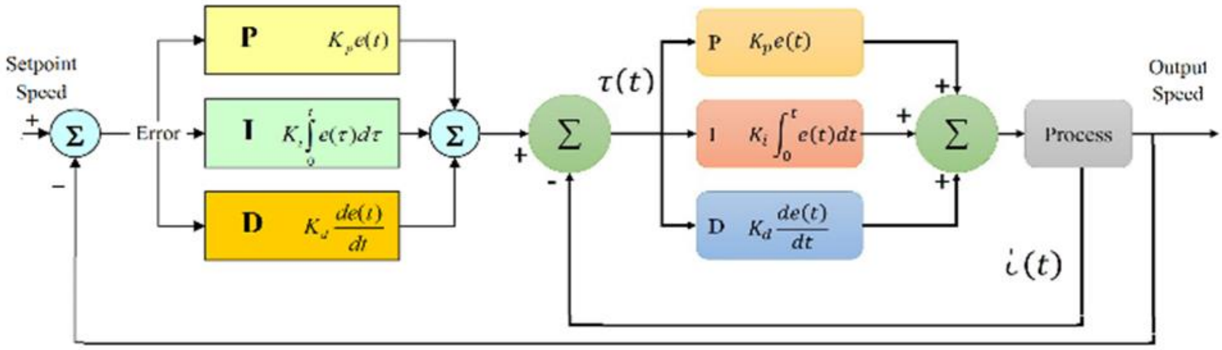

Fig. 4 Current and speed PID control schematic.

5.1 Auto-tuning method

Using feedback of the PID coefficient value for motor control can be theoretically obtained through formulas after modeling the motor system. there is. However, due to the nature of the airfoil bearing, frequent experiments to obtain the PID coefficient value can reduce the durability of the airfoil bearing because repetitive driving occurs in a stationary state or at low speed. In this paper, Ziegler Nichols' closed-loop method, which is an auto-tuning method, was used to identify the system characteristics and determine the PID coefficient with the first one-time drive. Ziegler Nichols' closed-loop method determines the PID coefficient value by identifying system characteristics from continuous output vibration [7].

To generate continuous output vibration, the relay-feedback method is applied. The feedback can generate continuous vibration from the error between the reference point and the output. The PID coefficient value can be determined by obtaining the ultimate coefficient from the generated vibration and applying it to the Ziegler-Nichols PID coefficient Table 1. The current PID coefficient value, which does not require rotation of the motor, was obtained using MATLAB/ SIMULINK, and the speed PID coefficient value was obtained through self-tuning in MATLAB.

Table 1: Ziegler-Nichols PID coefficients

	K_p	K_i	K_d
P	$0.6K_p$	-	-
PI	$0.4K_p$	$1.4K_p/T_o$	-
PID	$0.65K_p$	$2.2K_p/T_o$	K_pT_o

5.2 Self-tuning method using fuzzy controller

Airfoil bearing BLDC rotates at ultra-high speed with little friction with the bearing when the airfoil bearing operates in a normal state, and the characteristics of the load torque change nonlinearly as the motor speed increases. Therefore, it is necessary to perform adaptive control by self-tuning the PID coefficient values acquired through auto-tuning in real time. As the self-tuning method applied in this paper, the most commonly used fuzzy control was applied. The block diagram of the entire system combining the feedback method and the fuzzy controller is shown in Fig. 5.

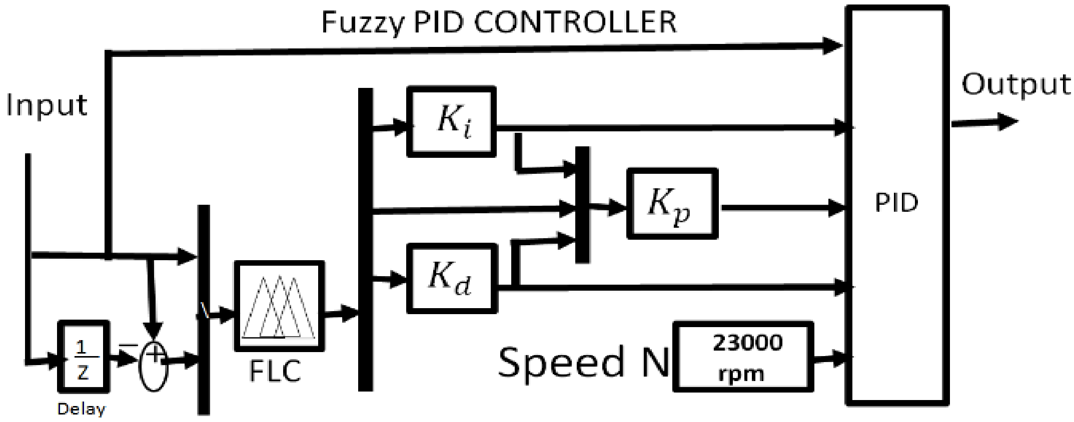

Fig. 5 The tuning of PID controller using fuzzy control

The input of the fuzzy controller is the speed error and the change rate of the error, and the output is, K_i, the PID coefficient adjustment value. The final velocity PID coefficient value to which the output of the fuzzy controller is applied is updated in real time to a value obtained by adding the output of the fuzzy controller to the previous coefficient value as shown in Equation (2) K_i, p K_d as shown in Fig. 6, the fuzz controller consists of fuzzification, rule base, inference, and defuzzification [8]. Fuzzification is a process of defining a fuzzy set by a membership function when a numerically expressed crisp value is input. Membership functions usually have the shape of a triangle or natural logarithm [16].

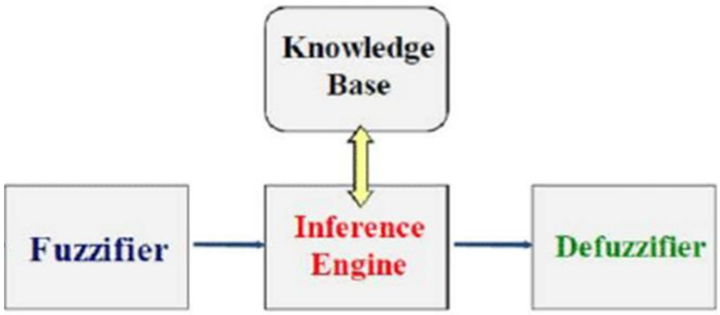

Fig. 6 The fuzzy controller consists of fuzzification, rule base, inference, and defuzzification

The triangular membership function was used, which is simple to implement and requires little computation [17]. In the inference process, as shown in Figure 7, the final membership function result can be obtained by "ANDing" the membership function output value for each rule obtained through the MAX-MIN operation. A fuzzy rule is defined as a result set of two or more combinations of antecedent sets and outputs [18].

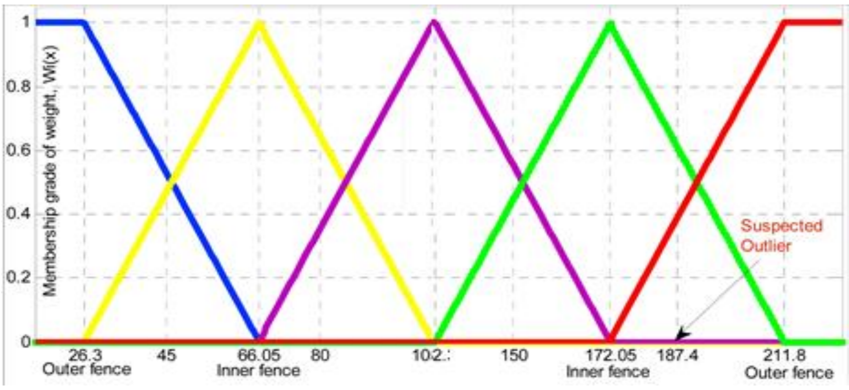

Fig. 7 The triangular membership function

For defuzzification, the commonly used Center of Area (COA) method was used [18]. as shown in black aria in Fig. 8.

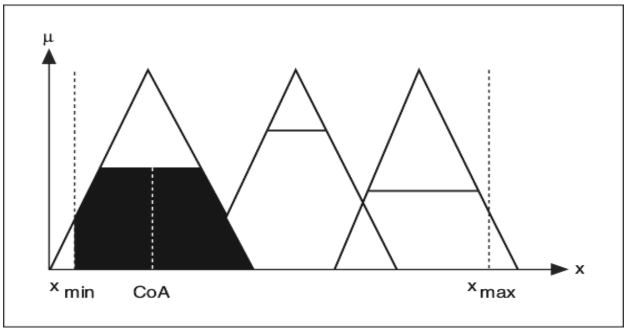

Fig. 8 Center of Area (COA) method

All processes of the fuzzy controller are operated by the rule base. The rule base is defined by knowledge and experience of the system. In this paper, rules for PID coefficient values must be defined, so each rule must be determined by the characteristics of PID coefficients. The system response characteristics according to the increase in the PID coefficient value in the control system are shown in Table 2. As shown in Table 2, considering the characteristics of the system response to the increase in the PID coefficient value. The rules are defined as shown in Table 3. Since the error is the largest in the control start section, the response at this time must reach the target value quickly, so the value of the proportional coefficient K must be changed larger. Similarly, the integral and differential coefficients also consider the characteristics of p-angular coefficients, and define the rule base as shown in Table 4.

Table 2. Impact of increasing PID coefficients

	Rise Time	Max.Peak Overshoot	Settling time	Steady state error
P	Decrease	Increase	Small Change	Decrease
I	Decrease	Increase	Increase	Small Change
D	Small Change	Decrease	Decrease	No Change

Table 3. System controller at Best values of PID

	First	Middle	Final
P	Very Large	Middle	Small
I	Small	Middle	Large
D	Small	Depend on stability and error	

6. System Description

Speed, current, and torque were measured at a DC input voltage of 24V. When operating at maximum speed, 40,000 rpm operation is possible, and when torque is applied, it satisfies the (torque τ = 0.03 Nm) at high speed (N=25000 rpm). In Fig. 3, the current voltage input and output of each phase must be measured to determine the efficiency. It is judged at a specify target because it exhibits the characteristics of (efficiency η=92%) at a speed of (N_r=23.2 rpm) and a torque of 0.025 Nm. The Motor parameters in table 5

Table 4. Rule bases for each PIO coefficient.

(a) Rule base of P coefficient.

ec/e	NB	NM	NS	Z	PS	PM	PB
NB	PB	PB	PM	PS	PS	Z	Z
NM	PB	PM	PS	PS	Z	Z	Z
NS	PM	PS	PS	Z	Z	Z	NS
Z	PS	PS	Z	Z	Z	NS	NM
PS	PS	Z	Z	Z	NS	NM	NM
PM	Z	Z	Z	NS	NM	NM	NB
PB	Z	Z	NS	NM	NM	NB	NB

(b) Rule base of I coefficient.

ec/e	NB	NM	NS	Z	PS	PM	PB
NB	NB	NB	PM	PS	PS	NM	NM
NM	NB	PM	NM	NM	NM	Z	Z
NS	NM	NM	NM	Z	Z	Z	PS
Z	NM	NM	NM	Z	PS	PS	PM
PS	PS	Z	Z	Z	PS	PM	PM
PM	Z	Z	PS	PS	PM	PM	PB
PB	Z	PS	PS	PM	PM	PB	PB

(c) Rule base of D coefficient.

ec/e	NB	NM	NS	Z	PS	PM	PB
NB	NB	NB	PM	Z	Z	PS	PS
NM	NB	PM	Z	PS	PS	Z	Z
NS	NM	Z	Z	Z	Z	Z	Z
Z	Z	PS	PS	Z	PS	PS	PM
PS	PS	Z	Z	Z	PS	PM	PM
PM	Z	Z	PS	PS	PM	PM	PB
PB	Z	PS	PS	PM	PM	PB	PB

Table 5. BLDC motor parameters with mounted airfoil bearing.

Parameters	Values
Rated voltage DC	415 Volt
Rated power	30 Hp
Stator resistor	37Ω
Stator inductance	0.41 mH

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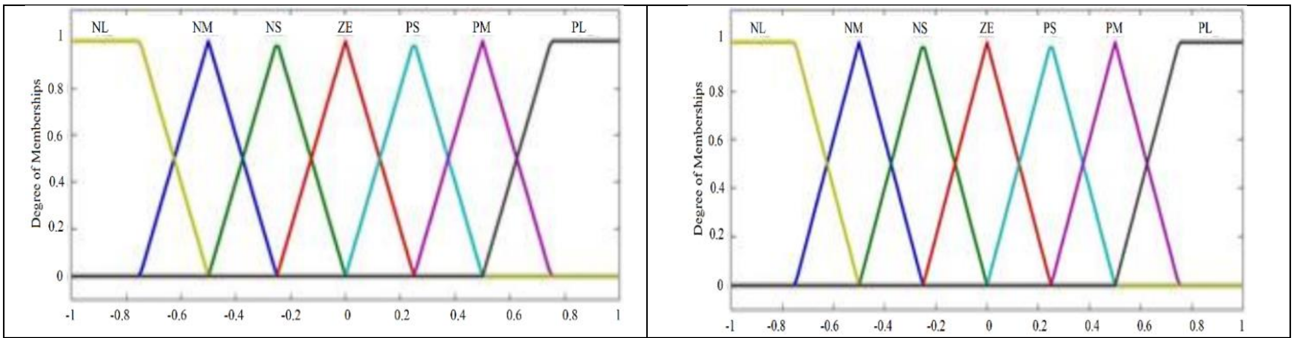
7. Consequences

In order to control ultra-high-speed BLDC motors equipped with airfoil bearings, it is important to understand the characteristics of airfoil bearings. When a certain torque is supplied to the motor, an air layer is formed between the rotating shaft and the airfoil bearing at a specific speed and operates in a normal state. As the load torque rapidly decreases in steady state, the system characteristics change significantly. The feedback method, one of the auto-tuning methods, was used to determine the PID coefficient value by identifying the system characteristics during normal operation. The relay-feedback method determines the PID coefficient value through system analysis from continuous output oscillation. BLDC motors equipped with airfoil bearings have different load torque depending on the speed.

Therefore, adaptive PID control was performed by combining the self-tuning method with the auto-tuning method. In this paper, the PID coefficient values are updated in real time by combining a fuzzy controller, one of the self-tuning methods. As a result of the experiment, it was confirmed that when controlled by combining the relay-feedback method and the fuzzy controller, the control is more stable in response to speed changes than when driven only with the PID coefficients obtained using the relay-feedback method. In this paper, airfoil A system that can stably control ultra-high-speed BLDC motors equipped with bearings has been established, and it can be applied to devices that operate BLDC motors equipped with airfoil bearings at ultra-high speeds in the future. As a future research task, it is necessary to study the case where the load is changed by attaching the impeller. In addition, research related to the dynamo system for ultra-high-speed operation is need.

8. Simulation Results

The memberships of FLC in Fig. 5: Two inputs (error e & change error ce) their memberships are shown in Fig. 9. Whereas, Fig. 10 shows the output memberships. Fig. 11 (a, b and c) shows the results of fuzzy surface of PID controller. Fig. 12. shows the comparison between the result of PID control was tuned by FLC (Fuzzy) and fixed PID control (non-fuzzy). Table 6. Shows the comparison table between the result of PID control was tuned by FLC (Fuzzy) and fixed PID control (non-fuzzy).



a) Memberships function for error

b) Memberships function for change of error

Fig. 9 Two inputs (error e & change error ce) of P I & D configuration

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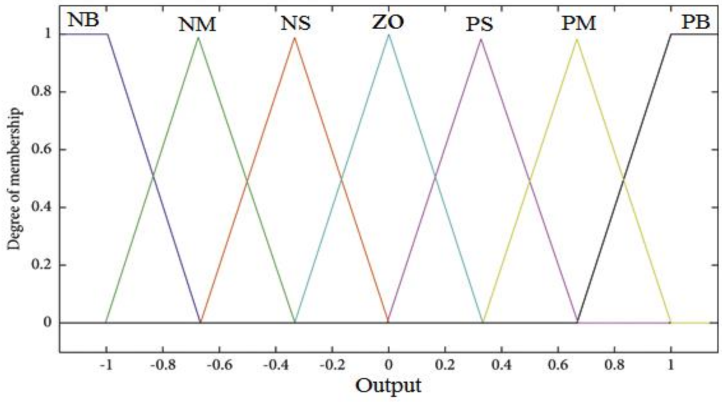
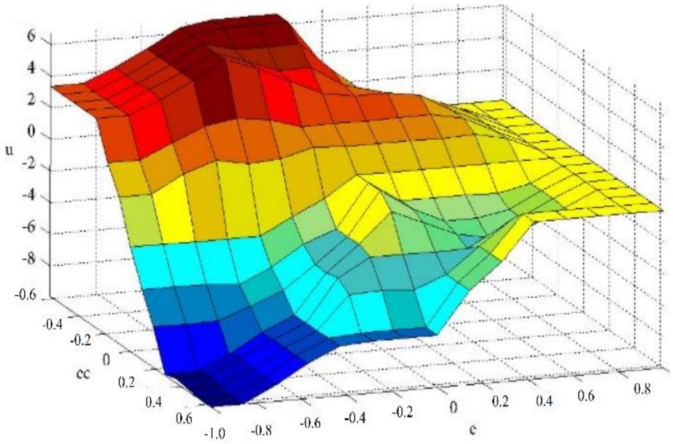
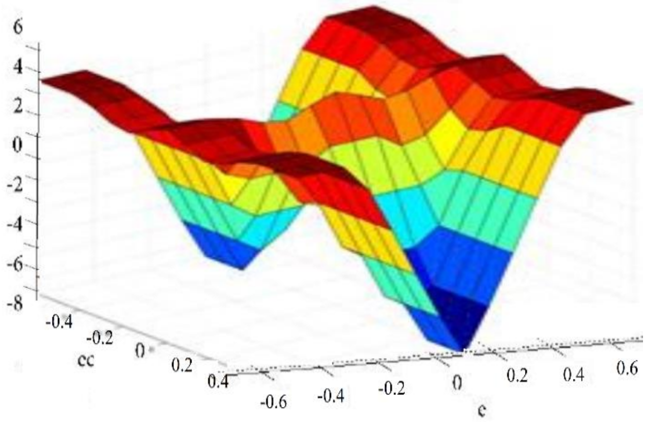


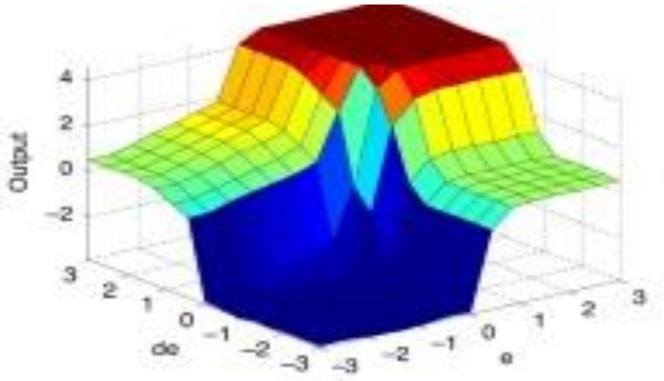
Fig.10 The output of P configuration



(a) Fuzzy surface of P coefficient.



(b) Fuzzy surface of I coefficient.



(c) Fuzzy surface of D coefficient.

Fig. 11 The Fuzzy surface of PID controller

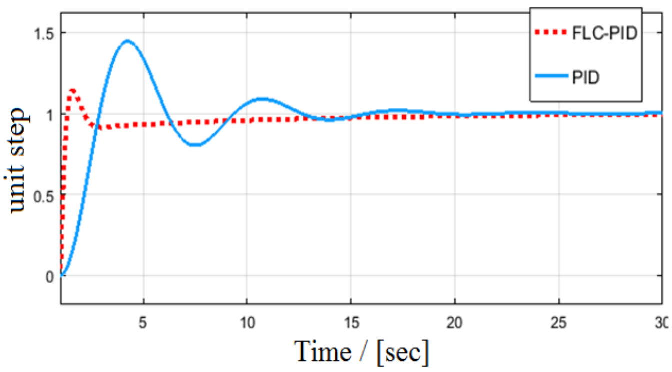


Fig.12 Comparison between the result of PID control was tuned by FLC (Fuzzy) and fixed PID control (non-fuzzy).

Table 6. Comparison table between the result of fuzzy control and non-fuzzy control.

	Non-Fuzzy	Fuzzy
Overshoot	3.974	1.57
Settling time	16.7 sec	6.54 sec

9. Conclusions

To control an ultra-high-speed BLDC motor equipped with an airfoil bearing, it is important to understand the characteristics of the airfoil bearing. When a certain torque is supplied to the motor, an air layer is formed between the rotating shaft and the airfoil bearing at a certain speed, and it operates in a normal state. In normal conditions, the load torque suddenly increases. As it decreases, the system characteristics change significantly. One of the auto-tuning methods was used to determine the PID coefficient value by identifying the system characteristics when operating in a normal state. The relay-feedback method determines the PID coefficient value through system analysis from continuous output vibration. BLDC motors equipped with airfoil bearings have different load torque depending on speed.

Therefore, adaptive PID control was performed by combining the self-tuning method with the auto-tuning method. In this paper, the PID coefficient value was updated in real time by combining a fuzzy controller, which is one of the self-tuning methods. When controlled by combining the method and the fuzzy controller, it was confirmed that the speed change was controlled more stable than when driven only with the PID coefficients obtained using the feedback method. In this paper, we have built a system that can stably control an ultra-high-speed BLDC motor equipped with an airfoil bearing, and in the future, it can be applied to devices that operate BLDC motors equipped with an airfoil bearing at ultra-high speed. As a future research project, research is needed on cases where the load changes by attaching an impeller. Additionally, research related to the dynamo system for ultra-high-speed operation is needed.

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